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COMMENT ON MCLENNAN AND SONNENSCHEIN "SEQUENTIAL BARGAINING AS A NONCOOPERATIVE FOUNDATION FOR WALRASIAN EQUILIBRIUM"

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TWO IMPORTANT ARTICLES in the literature on decentralized strategic markets are Gale (1986) and McLennan and Sonnenschein (1991). McLennan and Sonnenschein (1991) (henceforth, McLS) claim to have generalized Gale (1986) in several respects. In doing so, they introduce mainly two modifications: (i) unbounded short sales are allowed; (ii) agents do not observe their trading partner's characteristics. Within the framework of short sales, we show that feature (ii) is a crucial assumption for their results, as opposed to what the authors claim in the introduction (p. 1399, last paragraph): "...none of the results of Sections 2–4 are affected if agents are allowed to observe each other's utility functions and current bundles."

Specifically, Proposition 2 (p. 1419) and Theorem 3 (p. 1422) do not extend to the case where traders observe each other's characteristics, and it is an open question whether Theorem 2 does.

EXAMPLE 1: Imagine an economy that consists of a continuum of identical agents with strictly monotone preferences over a single commodity. Let each agent's endowment consist of one unit of the good. According to Proposition 2, the equilibrium expected utility of each agent corresponds to that associated with his optimal choice given the budget set induced by his current bundle and the unique supporting price established in Proposition 1. That is, in this example it should be the expected utility of consuming one unit of the good. However, consider the following strategies:

1. Each proposer who has less than two units asks to receive the rest of units he is short of two.
2. Each proposer who has two or more units proposes the zero trade.
3. Each responder who has less than two units accepts any trade that does not increase the proposer's bundle beyond two units, and also any trade that does not decrease his own bundle, and rejects otherwise.
4. Responders with at least two units accept only trades that increase their bundles, and leave otherwise.

It can be readily checked that these strategies constitute an equilibrium in the McLS model and that they implement an outcome where each agent ends up consuming two units of the good.

The reader may be puzzled by the above example, since every agent ends up consuming more than his initial endowments. However, as discussed in Dagan, Serrano, and Volij (1996), McLS's assumption of unlimited short sales implies that the feasibility

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constraints of the economy are often violated in the model, even in equilibrium. Proposition 2 applies to all equilibria, even to those that do not satisfy the feasibility constraints. We have not found a non-Walrasian example in which agents entering the economy sum up to a finite measure and the feasibility conditions are met. Therefore, it is still an open question whether the nonobservability of the partner's current bundle is required for Theorem 2, i.e., in strategic equilibria for which the feasibility condition H1(b) is satisfied.

It is also important to note that other models in the same context (such as Gale (1986)) are robust to the assumption of observability of trader's characteristics, and are constructed so that the feasibility constraints of the economy are respected in and out of equilibrium.

Based on the above counterexample to McLS's Proposition 2 in the case of the trading partner's bundle being observable, we construct a counterexample to their version of Theorem 3 for this case.

EXAMPLE 2: Consider an economy where in each period t a fixed positive measure of new agents enter the economy. Their preferences depend on their entry date (agents who enter in different dates have different preferences). Assume that agents observe their trading partner's utility function and current bundle. Modify the strategies of Example 1 so that all agents who enter in period t end up with $1 + 1/t$ units of the good. Every agent who enters the economy in period t when proposing in period $s \geq t$ asks for the units he is short of $1 + 1/t$ units, and so on. The reader can check that these strategies constitute an equilibrium in the McLS game. Note that the feasibility constraint H2(c), p. 1422, is met since the average amount of the good consumed is 1. However, every agent entering in period t consumes an amount $1 + 1/t > 1$, and therefore the strategies do not yield a Walrasian outcome. The outcome violates condition (γ) in McLS (p. 1405) by taking E to be the set of all utility functions and $w = 1$.

Andrew McLennan has pointed out to us that Example 2 may work without short sales. Indeed, it appears that we could make do with a modification of the above strategies so that responders who do not have enough to give to the proposer give him only those units they have. The reason this modification works is that the measure of entering agent is infinite, thereby implying that the probability of finding agents with enough units of the good is never negligible. On the other hand, we should stress that our Example 1 and McLS's Proposition 2 apply to the model when both the measure of entering agents is finite and infinite. In the former case, short sales and not infinite measures of agents are the perverse feature behind the "free lunch" aspects of the example. In order to avoid these infeasibility problems, several approaches are possible. One option is to rule out short sales and assume that the measure of entering agents is finite. This is the approach followed by Gale (1986), where by construction feasibility prevails both in and out of equilibrium. Alternatively, if one wishes to include unlimited short sales in the analysis, one should probably consider finite agent models, where infinite pyramid schemes cannot be created. Thus, the relevant question here is to look at the limit of such models, and we agree with McLS (p. 1402) that the equilibrium analysis of these models appears to be quite challenging.

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